Name of College - S. S. College J-Bad.

Deptt - Mathematics

Topic - Problem on Indeterminate Form

Class - B.Sc I (HONS)

Date - 17-06-2021

By Samrendra Kumar

Problem based on Form $0^0, \infty^0, 1^\infty$ Contd

Problem 1. $\lim_{x \to 1} x^{\frac{1}{1-x}}$

Let $y = \lim_{x \to 1} x^{\frac{1}{1-x}}$ [Form 1^∞]

$$\log y = \frac{1}{1-x} \log x$$

$$= \frac{\log x}{1-x}$$

$$\log y = \lim_{x \to 1} \frac{\log x}{1-x} \quad \left[\frac{0}{0}\right]$$

Applying L Hospital Rule

$$= \lim_{x \to 1} \frac{1}{x\,(-1)} = -1$$

$$\therefore \log y = -1$$

$$\Rightarrow y = e^{-1} = 1/e.$$

Problem 2.

$$\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}} \quad [1^{\infty}]$$

Let $y = \lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan \frac{\pi x}{2a}}$

$$\Rightarrow \lim_{x \to a} \log y = \lim_{x \to a} \tan \frac{\pi x}{2a} \log\left(2 - \frac{x}{a}\right) \quad [\infty \cdot 0]$$

$$= \lim_{x \to a} \frac{\log\left(2 - \frac{x}{a}\right)}{\cot \frac{\pi x}{2a}} \quad \left[\frac{0}{0}\right]$$

Applying L. Hospital Rule.

$$= \lim_{x \to a} \frac{1}{(2 - x/a)}\left(-\frac{1}{a}\right) \cdot \frac{1}{-\operatorname{cosec}^2 \frac{\pi x}{2a} \times \frac{\pi}{2a}}$$

$$= \lim_{x \to a} \frac{2}{\pi} \frac{\sin^2 \frac{\pi x}{2a}}{[2 - x/a]}$$

$$= \frac{2}{\pi} \cdot \frac{1}{1} = \frac{2}{\pi}$$

thus $\log y = \frac{2}{\pi} \Rightarrow y = e^{2/\pi}$

Problem 3. Evaluate

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$$

Let $y = \lim_{x\to 0}\left[\frac{\sin x}{x}\right]^{\frac{1}{x^2}} \quad [1^\infty]$

$$\log y = \lim_{x\to 0} \frac{1}{x^2}\log\frac{\sin x}{x}$$

$$= \lim_{x\to 0} \frac{\log \sin x - \log x}{x^2} \quad \left[\frac{0}{0}\right] \quad [\log^{m/n} - \log$$

Applying L-Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{1}{\sin x}\cdot \cos x - \frac{1}{x}}{2x}$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{x\cos x - \sin x}{x^2 \sin x} \quad \left[\frac{0}{0}\right]$$

$$= \frac{1}{2}\lim_{x\to 0} \frac{\cancel{\cos x} - x\sin x - \cancel{\cos x}}{x^2\cos x + 2x\sin x}$$

$$= -\frac{1}{2}\lim_{x\to 0} \frac{\sin x}{x\cos x + 2\sin x} \quad \left[\frac{0}{0}\right]$$

$$= -\frac{1}{2}\lim_{x\to 0} \frac{\cos x}{\cos x - x\sin x + 2\cos x}$$

$$= -\frac{1}{2}\,\frac{\cos 0}{3\cos 0} = -\frac{1}{6}$$

$$\therefore y = e^{-1/6}$$

Problem 4

Evaluate $\lim_{x \to 0} \left[\frac{\tan x}{x}\right]^{\frac{1}{x}}$

Let $y = \lim_{x \to 0} \left[\frac{\tan x}{x}\right]^{1/x}$ $[1^\infty]$

$$\Rightarrow \log y = \lim_{x \to 0} \frac{1}{x}\left[\log \tan x - \log x\right]$$

$$= \lim_{x \to 0} \frac{\log \tan x - \log x}{x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\tan x}\sec^2 x - \frac{1}{x}}{1}$$

$$= \lim_{x \to 0} \frac{\cos x}{\sin x} \times \frac{\sec x}{\cos x} - \frac{1}{x}$$

$$= \lim_{x \to 0} \frac{1}{\sin x \cos x} - \frac{1}{x}$$

$$= \lim_{x \to 0} \frac{2}{\sin 2x} - \frac{1}{x}$$

$$= \lim_{x \to 0} \frac{2x - \sin 2x}{x \sin 2x}$$

$$= \lim_{x \to 0} \frac{2 - 2\cos 2x}{\sin 2x + 2x \cos 2x} \quad \left[\frac{0}{0}\right]$$

$$= \lim_{x \to 0} \frac{4 \sin 2x}{2\cos 2x + 2\{\cos 2x - 2x \sin 2x\}}$$

$$= \frac{4 \cdot 0}{2 + 2} = 0 \qquad \therefore y = e^0 = 1$$